Transformation of irregular waves in the inner surf zone

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Abstract

A numerical model based on a high order non-oscillatory MacCormack TVD scheme is presented for the solution of the nonlinear Saint Venant equations in the inner surf zone. A comparison with measurements of surface elevations on a beach (Truc Vert Beach, France), shows that the model is capable of predicting the transformation of irregular waves in the inner surf zone and in particular the merging of broken waves.

1 Introduction

As the waves propagate shoreward to gradually smaller depth, their height and their steepness increase. This generally leads to the wave-breaking phenomenon, which is associated with strong turbulent motions. On gently sloping beaches we usually observe that after wave-breaking, the wave field reorganizes itself into borelike waves. This region has been termed the "inner surf zone" (ISZ) (see Svendsen (1984)). Lin and Liu (1998) have shown, from numerical simulations of Navier Stokes equations, that the pressure distribution under these waves is almost hydrostatic and that the horizontal velocity, excepted in the roller region, is nearly uniform over the depth. Thus, we can use the non-linear Saint Venant (SV) equations to describe the propagation and run-up of these broken waves (e.g. Hibbert and Peregrine (1979)). Kobayashi, DeSilva and Watson (1989) and Raubenheimer, Guza and Elgar (1996) have compared SV numerical simulations with respectively laboratory and field measurements in the ISZ and found a good agreement.

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In this paper, we theoretically analyze the ability of the inviscid SV equations to reproduce broken wave dissipation. Then, we present the development of a numerical model for solving the SV equations, based on a Mac Cormack TVD scheme, which is an efficient shock-capturing method. Finally, we show the ability of this numerical model to describe the irregular wave transformation along a transect spanning the ISZ on a beach. Moreover, we show the importance of the phenomenon of merging of broken waves in the evolution of an irregular wave field.

2 Shock conditions and energy dissipation in bores

The flow in broken waves being turbulent, the inviscid SV theory is questionable in determining the energy dissipation. In this section, we present a theoretical analysis based on the "weak solution" concept, to evaluate the ability of these equations to reproduce broken wave dissipation.

The nonlinear SV equations can be expressed in conservation form as:

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}(hu) = 0 \tag{1}$$

$$\frac{\partial}{\partial t}(hu) + \frac{\partial}{\partial x}(hu^2 + \frac{1}{2}gh^2) = gh\frac{\partial d}{\partial x} - \frac{\tau_b}{\rho} \tag{2}$$

where $u(x,t)$ is the depth-averaged horizontal velocity, $h(x,t)$ is the total water depth, $d(x)$ is the still water depth, g is the gravitational acceleration and ρ is the fluid density. The bottom shear stress may be expressed as $\tau_b = \frac{1}{2}\rho f|u|u$, where f is the constant bottom friction coefficient. As shown by Kobayashi and Wurjanto (1992) and Cox et al. (1994) the wave energy dissipation is caused predominantly by wave breaking and the effect of bottom friction is essentially limited to the swash zone. Thus, in the following theoretical discussion τ_b will be neglected.

Following the concept of "weak solutions" (Whitham (1974)) we can approximate wave fronts, which develop in the ISZ, by introducing discontinuities satisfying shock conditions based on mass and momentum conservation across the shock:

$$-U[h] + [hu] = 0$$
$$-U[hu] + [hu^2 + \frac{1}{2}gh^2] = 0$$

where the brackets indicate the jump in the quantity and U is the shock velocity. Using subscript 1 for values ahead of the shock and subscript 2 for values behind, the shock conditions may also be written in the form given by:

$$u_1 - U = \epsilon\left(\frac{gh_2}{2h_1}(h_2+h_1)\right)^{\frac{1}{2}} \qquad (\epsilon = \pm 1)$$
$$u_2 - U = \epsilon\left(\frac{gh_1}{2h_2}(h_2+h_1)\right)^{\frac{1}{2}}$$

Two solutions are possible depending on the sign of ϵ, but only one physical solution exists. To clarify this point it is necessary to study the energy balance across a shock. In continuous parts of the flow, a conservation equation for the energy $\mathcal{E}$ can be derived from equations (1) and (2):

$$\frac{\partial \mathcal{E}}{\partial t} + \frac{\partial \mathcal{F}}{\partial x} = 0$$

where, $\mathcal{E} = \frac{1}{2}\rho(hu^2 + g(h-d)^2)$, $\mathcal{F} = \rho hu\ (\frac{1}{2}u^2 + g(h-d))$. However, when a shock occurs the energy is no longer conserved. The energy dissipation $\mathcal{D}$ across a shock is given by $\mathcal{D} = -[\mathcal{F}] + U[\mathcal{E}]$ (see Stoker (1957)), which leads to equation:

$$\mathcal{D} = -\epsilon\ \frac{\rho g}{4}(h_2 - h_1)^3\Big(\frac{g(h_2+h_1)}{2h_1h_2}\Big)^{\frac{1}{2}}$$

Finally, if we postulate that the particles crossing the shock must lose energy ($\epsilon = -1$), the shock conditions are:

$$u_1 - U = -\Big(\frac{gh_2}{2h_1}(h_2+h_1)\Big)^{\frac{1}{2}} \tag{3}$$

$$u_2 - U = -\Big(\frac{gh_1}{2h_2}(h_2+h_1)\Big)^{\frac{1}{2}} \tag{4}$$

and the energy dissipation is given by:

$$\mathcal{D} = \frac{\rho g}{4}(h_2 - h_1)^3\Big(\frac{g(h_2+h_1)}{2h_1h_2}\Big)^{\frac{1}{2}} \tag{5}$$

So, when bores are involved, energy is not conserved. In the present formulation the basic flow is inviscid, but dissipation is implicit in the shock representation. In the coastal engineering literature (e.g. Battjes and Janssen (1978)), the equation (5) is usually obtained for bore connecting two regions of uniform flow (hydraulic jump). It is important to stress that in fact this equation can be applied to any wave front. Bonneton (2000), using this approach, developed an analytical model based on a nonlinear hyperbolic theory to describe distortion and dissipation related to the propagation of periodic waves on a gently sloping bottom. The good agreement between this analytical model and wave flume experiments performed by Stive (1984) showed that equation (5) gives a good estimate of the energy dissipation. For complex beach profiles and irregular waves the solution of equations (1) (2) (3) and (4) requires to use numerical methods.

3 Time-Dependent numerical model

To compute broken wave propagation it is necessary to use a shock-capturing numerical method. Hibberd and Peregrine (1979) and Kobayashi et al. (1989) have chosen the Lax-Wendroff scheme, which has been successfully applied for solving numerous hyperbolic systems. However, in presence of fronts, the dispersive properties of this scheme introduce spurious numerical oscillations. To reduce these high-frequency oscillations which tend to appear at the rear of the wave front, Hibberd and Peregrine (1979) and Kobayashi et al. (1989) included an additional dissipative term. However, in some cases (see Kobayashi et al. (1989,1992)) numerical oscillations are still present. An alternative to this approach is to use a TVD (total variation diminishing) scheme, which represents a rational method for the determination of artificial dissipation terms. The SV equations are solved using the MacCormack scheme with a TVD flux correction in order to make it non-oscillating in presence of wave fronts.

SV equations can be expressed in vectorial form as:

$$\frac{\partial \mathbf{q}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} = \mathbf{S} \tag{6}$$

$$\mathbf{q} = \begin{pmatrix} h \\ hu \end{pmatrix}, \; \mathbf{F} = \begin{pmatrix} hu \\ hu^2 + \frac{g}{2}h^2 \end{pmatrix}, \; \mathbf{S} = \begin{pmatrix} 0 \\ gh\frac{\partial d}{\partial x} - \frac{1}{2}f|u|u \end{pmatrix}$$

Let $\mathbf{q}_i^n$ be the numerical solution of equation (6) at $x = i\Delta x$ and $t = n\Delta t$, with Δx the spatial mesh size and Δt the time step; $\lambda = \Delta t/\Delta x$. The TVD MacCormack scheme can be expressed in three steps:

(a) predictor step

$$\mathbf{q}_i^1 = \mathbf{q}_i^n - \lambda(\mathbf{F}_{i+1}^n - \mathbf{F}_i^n) + \Delta t \mathbf{S}_i^n$$

(b) corrector step

$$\mathbf{q}_i^2 = \frac{1}{2}[\mathbf{q}_i^1 + \mathbf{q}_i^n - \lambda(\mathbf{F}_i^1 - \mathbf{F}_{i-1}^1) + \Delta t \mathbf{S}_i^1]$$

(c) TVD step

$$\mathbf{q}_i^{n+1} = \mathbf{q}_i^2 + \frac{\lambda}{2}(R_{i+\frac{1}{2}}^n \mathbf{\Phi}_{i+\frac{1}{2}}^n - R_{i-\frac{1}{2}}^n \mathbf{\Phi}_{i-\frac{1}{2}}^n)$$

where the notation $i+\frac{1}{2}$ corresponds to quantities estimated at the mesh interface $(i, i+1)$. R is the right-eigenvector matrix of the flux Jacobian matrix $A = \dfrac{\partial \mathbf{F}}{\partial \mathbf{q}}$ and the l^{th} component of the vector $\mathbf{\Phi}_{i+\frac{1}{2}}^n$ is defined by:

$$(\phi_{i+\frac{1}{2}}^l) = \frac{1}{2}[|a_{i+\frac{1}{2}}^l| - \lambda(a_{i+\frac{1}{2}}^l)^2](\alpha_{i+\frac{1}{2}}^l - Q_{i+\frac{1}{2}}^l)$$

where $a_{i+\frac{1}{2}}^l$ represents the l^{th} component of the vector of local characteristic speeds, $\alpha_{i+\frac{1}{2}} = R_{i+\frac{1}{2}}^{-1}(\mathbf{q}_{i+1} - \mathbf{q}_i)$, and $Q_{i+\frac{1}{2}}^l = minmod(\alpha_{i-\frac{1}{2}}^l, \alpha_{i+\frac{1}{2}}^l, \alpha_{i+\frac{3}{2}}^l)$ is the flux limiter. At the interface of the meshes $\mathbf{q}_{i+1/2}$ is determined by Roe's averaging, which is an approximate Riemann solver.

The steps (a) and (b) describe the classical MacCormack scheme, whereas the step (c) is a TVD flux correction which has been described in detail by Vincent et al. (2000). The TVD MacCormack scheme so obtained retains second-order precision in space and time in regular zones and is oscillation-free across wave fronts.

At the seaward boundary we have implemented a method developed by Cox et al. (1994), which determines the outgoing Riemann invariant by an implicit scheme. This method allows to specify the measured water depth $h(t)$ directly at the seaward boundary of the domain ($x = 0$). To manage the swash zone evolution we impose in dry meshes a thin water layer $h_{min} = 10^{-4}m$, with $u = 0$. Thus, SV equations are solved everywhere in the computational domain. However, at the shoreline (wet meshes next to dry meshes) a specific treatment is applied in the momentum equation to the discretization of the horizontal gradient of the surface elevation ($gh\frac{\partial \eta}{\partial x} = gh(\frac{\partial h}{\partial x} - \frac{\partial d}{\partial x})$, where η is the surface elevation). It consists in omitting the landward spatial differences of this term in the predictor and corrector steps. Vincent et al (2000) have shown that this simple numerical treatment gives very accurate results in describing shore-line evolution for a non-breaking wave climbing a beach.

The present numerical model does not include explicitly a physical representation of the energy dissipation due to wave breaking. The modelling of broken wave dissipation is given by the numerical dissipation of our shock-capturing numerical scheme in presence of wave fronts. Bonneton et al. (1999) have shown that if Δx is small enough to describe the wave front, the computed dissipation is not sensitive to the spatial resolution. For instance, they have simulated the propagation of a wave packet on a planar bottom in a periodic domain with no friction and a constant water depth. When a bore is formed, the energy decrease becomes independent of the spatial resolution and followed a $t^{-1/2}$ law, which can predicted by the nonlinear hyperbolic theory developed by Bonneton (2000).

4 Comparison with field data

In this section the numerical model is compared with the field experiments carried out in may 1998 at Truc Vert Beach (Sénéchal et al. (2000)). It is a sandy beach, located 5 km North of the Arcachon lagoon inlet (SW France), in a mesotidal context. In this paper, we analyze one data run of this fieldwork, corresponding to pressure measurements collected near high tide. Three pressure sensors were deployed on a cross-shore transect (see figure 1), in mean depths between 0.3m and 1.3m. In figure 1, $z = 0$ corresponds to the mean level of the surface elevation at $x = 0$ (P1 location). Data runs were acquired at a 3 Hz sample rate. Water depths were estimated assuming that the measured pressure field is hydrostatic. Offshore waves were irregular and characterized by a broad band spectrum with a spectral peak period of $6s$ (see figure 2). Their significant wave height was 0.85m and the wave direction was approximately normal to the beach.

The computations are performed using $\Delta x = 0.15m$ and, following calibrations done by Raubenheimer et al. (1995), a friction coefficient $f = 0.015$. Com-

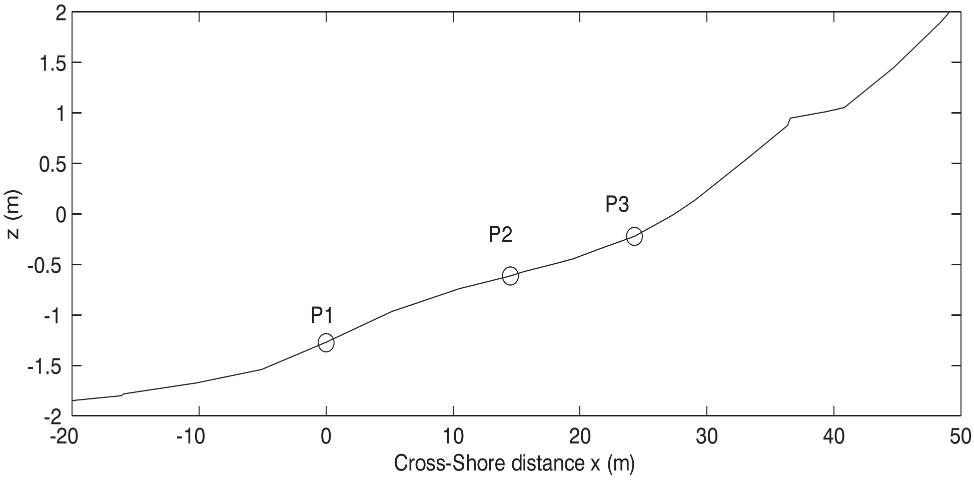

Figure 1: Cross-shore bathymetry for Truc Vert beach and locations of pressure sensors (circles).

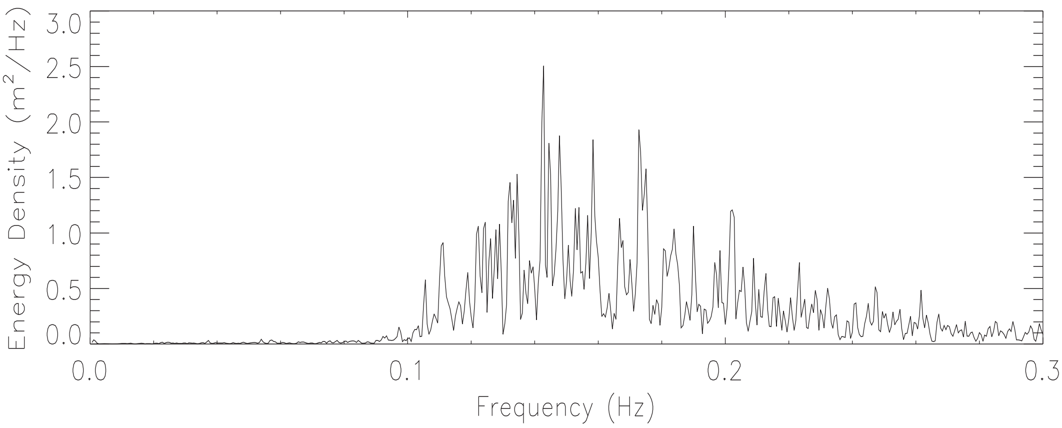

Figure 2: Offshore sea surface elevation energy density spectra in 8 m water depth.

parison of model predictions using a smaller mesh size ($\Delta x = 0.024m$) shows that predicted time series are not significantly affected by Δx (excepted for the wave front steepness). The duration of the run was $900s$ ($-50 < t < 850$), where $t = -50s$ is the start of the computation. The initial condition of no wave motion leads to a transient period $t \in [-50, 0]$, which is eliminated from time series presented hereafter. The seaward boundary condition of the model is given by time series of water depth (P1 sensor), corresponding to the shoreward propagation wave field.

In a previous paper, Bonneton et al. (1999) have shown that the observed decrease of wave height and increase of wave asymmetry as waves propagate shoreward were well predicted by the model. Figure 3 shows the predicted and observed cross-shore evolution of the sea surface elevation energy density spectra. The observed increase of infragravity energy and decrease of sea-swell energy are well predicted. However we note that at low frequency ($f < 0.04$ hz) the model slightly overestimates the energy. This discrepancy could be due to longshore effects which are not taking into account in our model. We note a very good agreement between model predictions and observations of the energy decrease at sea-swell frequencies. This demonstrates that the shock representation presented in section 2 is relevant to describe broken wave dissipation.

Measured and computed time series of local water depth h at cross-shore locations P1, P2 and P3, are shown in figure 4. We observe that the number of wave fronts decreases from 25 in P1 to 15 in P3. This is related to the merging of broken waves. For instance, waves n° 1, 3 and 5 are so slow that they get increasingly closer to the following waves, respectively n° 2, 4 and 6. Then, when a bore overtakes another bore, they merge into a single bore. The agreement between the predicted and observed evolution of local water depth shows the ability of the inviscid nonlinear SV equations to describe the merging of waves.

5 Conclusion

After some theoretical clarifications about energy dissipation in bores, a new model that solves the non-linear Saint Venant equations with a high order MacCormack TVD scheme has been presented in this paper. This model is able to describe broken wave propagation without introducing numerical oscillations at the rear of the wave front. The numerical model is shown to accurately predict the irregular wave transformation observed along transects spanning the ISZ on Truc Vert Beach. We have shown that the merging of broken waves plays an important role in the transformation of an irregular wave field. The agreement between the predicted and observed evolution of the wave field demonstrates that this phenomenon is well described by the inviscid SV equations. This phenomenon should be further investigated experimentally and theoretically on the basis of the nonlinear hyperbolic theory of shock wave.

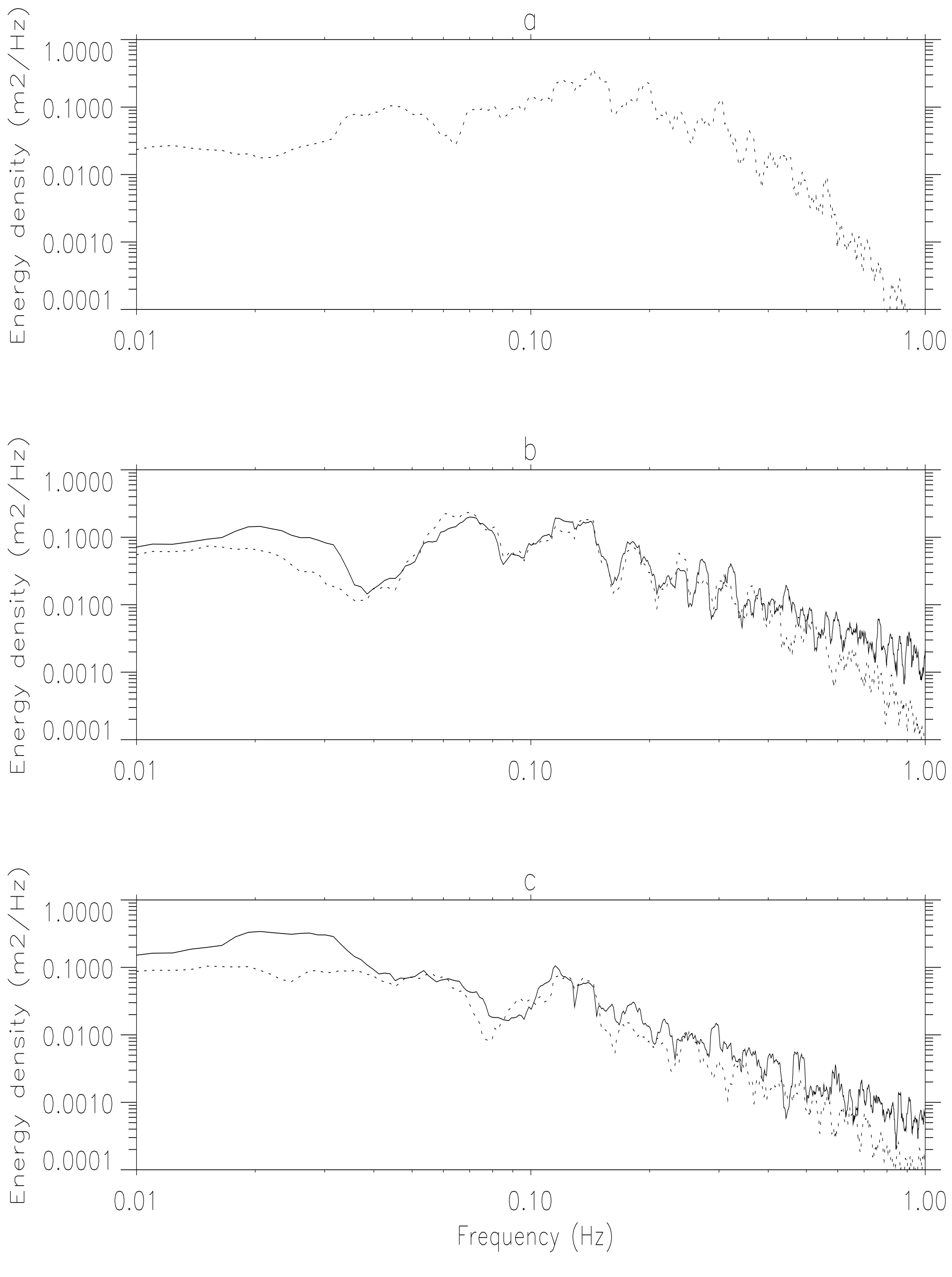

Figure 3: Measured (dashed line) and computed (solid line) sea surface elevation energy density spectra at cross-shore locations: (a), $x = 0$ (P1); (b), $x = 14.5m$ (P2); (c), $x = 24.5m$ (P3).

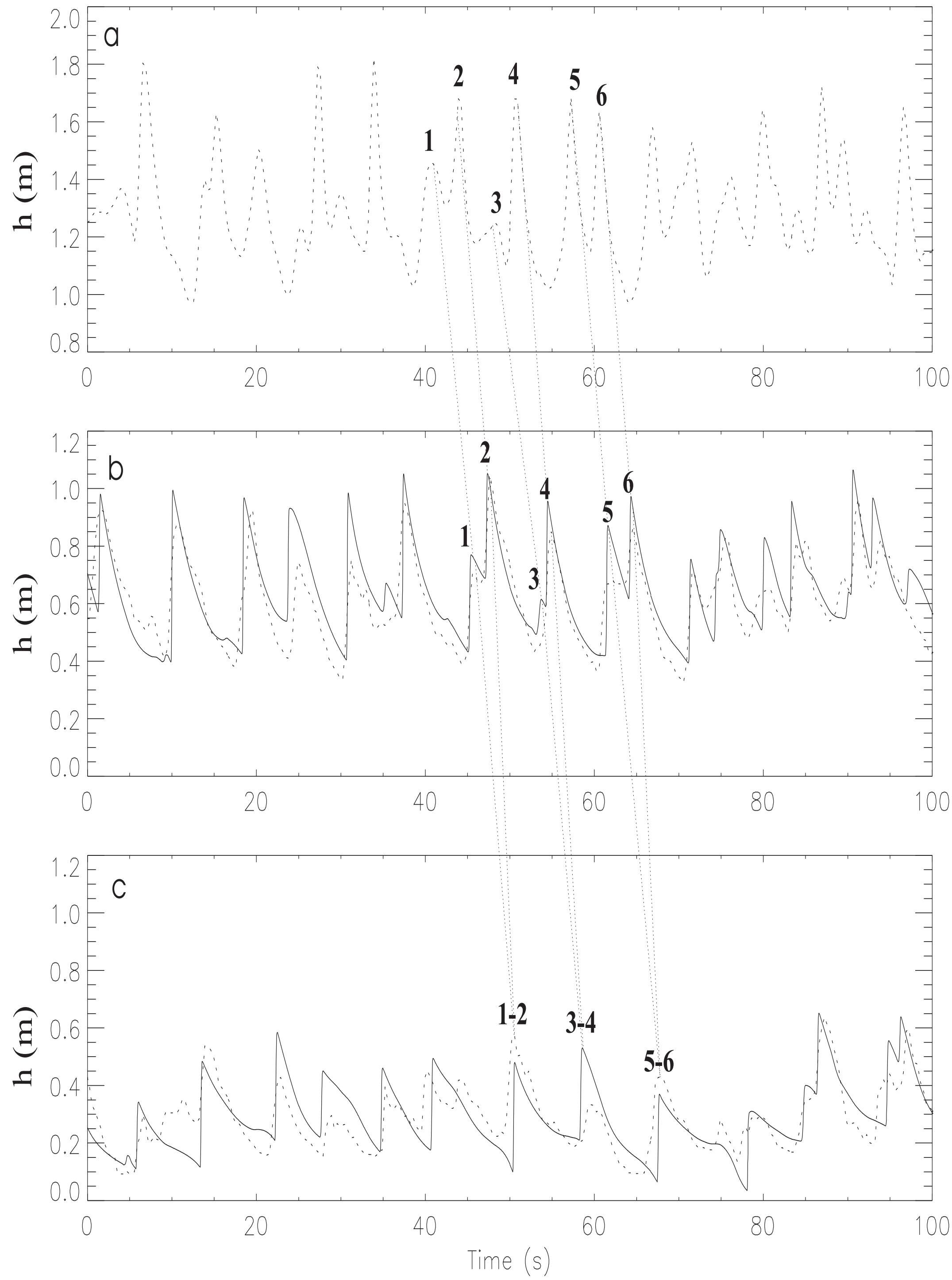

Figure 4: Measured (dashed line) and computed (solid line) time series of water depth h at cross-shore locations: (a), $x = 0$ (P1); (b), $x = 14.5m$ (P2); (c), $x = 24.5m$ (P3).

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